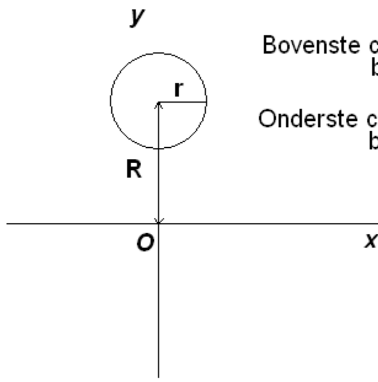

Bovenste cirkel boog = $y = R + \sqrt{r^2 - x^2}$

Onderste cirkel boog = $y = R - \sqrt{r^2 - x^2}$

Bovenste cirkelboog roteren om x-as en oppervlakte van omwentelingslichaam bepalen:

$$f(x) = R + \sqrt{r^2 - x^2}$$

$$f'(x) = \frac{-2x}{2\sqrt{r^2 - x^2}} = \frac{-x}{\sqrt{r^2 - x^2}}$$

$$\left(f'(x)\right)^2 = \frac{x^2}{r^2 - x^2}$$

$$2\pi \cdot \int \left(R + \sqrt{r^2 - x^2}\right)\left(\sqrt{1 + \frac{x^2}{r^2 - x^2}}\right) dx$$

$$= 2\pi \cdot \int \left(R + \sqrt{r^2 - x^2}\right)\left(\frac{r}{\sqrt{r^2 - x^2}}\right) dx$$

$$= 2\pi \cdot \int \left(R \frac{r}{\sqrt{r^2 - x^2}} + r\right) dx$$

$$= 2\pi \cdot \left(\int R \frac{r}{\sqrt{r^2 - x^2}} dx + \int r dx\right)$$

$$= 2\pi \cdot \left(Rr \int \frac{1}{\sqrt{r^2 - x^2}} dx + rx\right)$$

$$= 2\pi \left(Rr \cdot \ arcsin\left(\frac{x}{r}\right) + rx\right)$$

Dus $2\pi \int_{-r}^{r} \left(R + \sqrt{r^2 - x^2}\right)\left(\sqrt{1 + \frac{x^2}{r^2 - x^2}}\right) dx = 2\pi \left[Rr \cdot \ arcsin\left(\frac{x}{r}\right) + rx\right]_{x=-r}^{x=r}$

$= 2\pi\left(Rr \cdot \ arcsin(1) + r^2 - (Rr \cdot \ arcsin(-1) - r^2)\right) = 2\pi\left(\frac{1}{2}\pi \cdot R \cdot r + r^2 + \frac{1}{2}\pi \cdot R \cdot r + r^2\right)$

$= 2\pi(\pi \cdot R \cdot r + 2r^2)$

Onderste cirkelboog roteren om x-as en oppervlakte van omwentelingslichaam bepalen:

$$f(x) = R - \sqrt{r^2 - x^2}$$

$$f'(x) = \frac{2x}{2\sqrt{r^2 - x^2}} = \frac{x}{\sqrt{r^2 - x^2}}$$

$$\left(f'(x)\right)^2 = \frac{x^2}{r^2 - x^2}$$

$$2\pi \cdot \int \left(R - \sqrt{r^2 - x^2}\right)\left(\sqrt{1 + \frac{x^2}{r^2 - x^2}}\right) dx$$

$$= 2\pi \cdot \int \left(R - \sqrt{r^2 - x^2}\right)\left(\frac{r}{\sqrt{r^2 - x^2}}\right) dx$$

$$= 2\pi \cdot \int \left(R\frac{r}{\sqrt{r^2 - x^2}} - r\right) dx$$

$$= 2\pi \cdot \left(\int R\frac{r}{\sqrt{r^2 - x^2}} dx - \int r dx\right)$$

$$= 2\pi \cdot \left(Rr \int \frac{1}{\sqrt{r^2 - x^2}} dx - rx\right)$$

$$= 2\pi \left(Rr \cdot\ arcsin\left(\frac{x}{r}\right) - rx\right)$$

Dus $2\pi \int_{-r}^{r} (R - \sqrt{r^2 - x^2}) \left(\sqrt{1 + \frac{x^2}{r^2 - x^2}}\right) dx = 2\pi \left[Rr \cdot\ arcsin\left(\frac{x}{r}\right) - rx\right]_{x=-r}^{x=r}$

$= 2\pi\big(Rr \cdot\ arcsin(1) - r^2 - (Rr \cdot\ arcsin(-1) + r^2)\big) = 2\pi\left(\frac{1}{2}\pi \cdot R \cdot r - r^2 + \frac{1}{2}\pi \cdot R \cdot r - r^2\right)$

$= 2\pi(\pi \cdot R \cdot r - 2r^2)$

Dus totale oppervlakte omwentelingslichaam (van de torus dus) is

$2\pi(\pi \cdot R \cdot r + 2r^2)$ + $2\pi(\pi \cdot R \cdot r - 2r^2)$ = $2\pi(2 \cdot \pi \cdot R \cdot r + 2r^2 - 2r^2)$ = $\boldsymbol{4\pi^2 \cdot R \cdot r}$